An Efficient Compression Representation of Adaptive Cross-approximation for Analysis Microstrip Problems

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In order to efficiently solve large dense complex linear systems arising from electric field integral equations (EFIE) formulation of electromagnetic problems, the adaptive cross-approximation (ACA) has been used to accelerate the matrix-vector product (MVP) operations. This paper presents an efficient representation of impedance matrix of ACA for microstrip problems. First, a series of basis matrices at lowest level are constructed by singular value decomposition (SVD). Based on the basis matrices, the far-field interaction parts of impedance matrix can be represented by multiplication of three sparse matrices in recursive manner. Numerical experiments demonstrate that our efficient representation of ACA algorithm outperforms original ACA in terms of memory of far-field interactions and the time per MVP operation.

***Index Terms*—Computational electromagnetics, method of moments, compression algorithm**

I. Introduction

Efficient modeling and simulation of electromagnetic (EM) scattering and radiation from conducting structures continues to be an active topic of research. The method of moments (MoM) based on electric field integral equations (EFIE) [1] has been a very efficient computational method for the EM analysis of arbitrary shaped antennas. However, the conventional MoM results in a dense linear system to solve. Direct methods for inverting the dense impedance matrix require $O(N^3)$ operations. This necessity renders conventional MoM impractical for real-life applications.

In recent years, there have been a number of techniques that successfully reduce the memory requirements as well as the computational time associated with the iterative solution of EFIE. The multilevel fast multiple algorithm (MLFMA) [2] is famous and widespread in computational EM. The MLFMA reduces the numerical complexity of both memory and CPU time to $O(N\log N)$. However, MLFMA is the Green's function dependent, because it decomposes the Green's function through Gegenbauer's additional theorem. For complex Green's functions, MLFMA becomes much more involved than the case of the free space Green's function. The fast Fourier transformation (FFT) based methods such as the sparse-matrix canonical grid method(SMCG)[3] use a uniform grid to compute matrix entries so that a Toeplitz matrix is resulted where the FFT can be utilized to speed up the matrix-vector product (MVP). For the volume integral equation (VIE), the FFT methods achieve complexity of $O(N\log N)$, whereas for surface equation, the complexity is reduced to $O(N^{3/2})$.The adaptive cross approximation (ACA) [4, 5] algorithm has been developed and widely used to solve large magneto-static problems and electromagnetic wave problems of moderate electric size. In contrast with MLFMA, the ACA algorithm is purely algebraic and, therefore, does not depend on Green's function. Moreover, the great advantage of ACA algorithm is that it can be easily integrated into the existing MoM codes.

In this paper, we propose an efficient compression representation of impedance matrix of ACA algorithm firstly. Because of complex Green's function, the efficient compression representation of ACA algorithm is then used to analyze the microstrip problems. Numerical experiments demonstrate that the efficient compression representation of impedance matrix of ACA is more efficient than original ACA in terms of memory requirements and MVP time per iteration.

II. Method

The EFIE formulation of the analysis of microstrip problem using planar Rao-Wilton-Glisson (RWG) basis functions for surfaces modeling is presented in[1]. The resulting linear systems after Galerkin's testing reads

$$\mathbf{Z}\cdot\mathbf{I} = \mathbf{V} \tag{1}$$

where $\mathbf{Z}$ is the impedance matrix and is symmetrical, $\mathbf{I}$ is column vector containing the unknown coefficients of the surface current expansion with RWG basis functions and $\mathbf{V}$ is the discretization of the incident field.

In ACA algorithm, we use a hierarchical decomposition of the problem domain into an oct-tree of groups, which is the same as MLFMA. L is the level of the oct-tree. Let $\mathbf{Z}^{\mathrm{NF}}$ denote submatrices corresponding to the near-field field interaction, Z_l denotes the far-field interaction parts of the impedance matrix at level l. The impedance matrix can be represented as $\mathbf{Z}=\mathbf{Z}^{\mathrm{NF}}+\mathbf{Z}_L+\cdots+\mathbf{Z}_l+\cdots+\mathbf{Z}_2$. According to ACA algorithm, each submatrix $\mathbf{Z}_{ij}^l$ of Z_l can be represented as the product of two low rank matrices as follow $\mathbf{Z}_{ij}^l=\mathbf{U}_{ir}^l\mathbf{V}_{rj}^l$.

In this paper, an efficient compression representation of the ACA algorithmis proposed. At the lowest level L, the far-field interaction submatrix Z_l can be represented as

$$\mathbf{Z}(L) = \mathrm{diag}\left(\cdots \quad \mathbf{U}_{ir_i}^L \quad \cdots\right)\hat{\mathbf{Z}}_L\left[\mathrm{diag}\left(\cdots \quad \mathbf{U}_{jr_j}^L \quad \cdots\right)\right]^{\mathrm{c}} \tag{3}$$

where $\mathbf{U}_{ir_i}^L$ and $\hat{\mathbf{Z}}_L$ are basis and coupling matrices, respectively. At l(l>1) level, the basis $\mathbf{U}_{ir_i}^l$ is represented as $\mathbf{U}_{ir_i}^l = \mathbf{U}_i^{l+1}\mathbf{E}_i^{l+1}$, where $\mathbf{U}_i^{l+1}$ is basis corresponding to nonempty subgroup at level l+1 whose parent group is i, and $\mathbf{E}_i^{l+1}$ is transfer matrix at level l+1. Thus, the efficient compression representation of ACA algorithm can be obtained in a multilevel recursive manner as follows:

Procedure BuildMatrix

{ If (i is a nonempty group in the lowest level)

$\hat{\mathbf{Z}}_l' = 0$

For (i = 1: $m(l)$: i++) $\hat{\mathbf{Z}}_l'(i) = \mathbf{Z}_l|_i \xrightarrow{\text{SVD}} \mathbf{U}_l(i)$

$\mathbf{U}_l = \text{diag}(\mathbf{U}_l(1), \mathbf{U}_l(2), \cdots, \mathbf{U}_l(m(l)))$

$\hat{\mathbf{Z}}_l = \mathbf{U}_l \mathbf{U}_l^H \mathbf{Z}_l \bar{\mathbf{U}}_l \mathbf{U}_l^T$

Else

For (i = 1: $m(l)$: i++)

{ $\hat{\mathbf{Z}}' = \mathbf{Q}_l^H \mathbf{Z}_l \xrightarrow{\text{SVD}}$ transfer matrix $\mathbf{E}_l'$

Basis matrix: $\mathbf{U}_l = \mathbf{Q}_l \mathbf{E}_{l+1}$ }

For (i = 1: $m(l)$: i++) $\hat{\mathbf{Z}}_l = \mathbf{U}_l \mathbf{U}_l^H \mathbf{Z}_l \bar{\mathbf{U}}_l \mathbf{U}_l^T$

}

where $m(l)$ denotes the number of far-field interaction non-empty group of subgroup i at level l, and $\mathbf{Q}_l$ is corresponding to basis matrix at level l+1. Based on above procedure, the MVP operation is similar to MLFMA. The efficient compression of ACA algorithm is referenced as ACA-CR for simplicity.

III. Results

A. The validation of ACA-CR

To validate the ACA-CR algorithm, a microstrip circuit is analyzed in Fig. 1. The parameters of the microstrip are W=10.08mm, L=11.79mm, d_1=3.93mm, d_2=1.3mm, L_1=14.6 mm, L_2=13.4mm, L_3=12.32mm. The relative permeability ε_r and the thickness of the dielectric substrate are 2.1 and 1.5748mm, respectively. The number of unknowns is 400. The radiation pattern of the microstrip is displayed in Fig. 2. It can be seen that the results of ACA-CR agree with the measurement in [6] very well. This validates the ACA-CR.

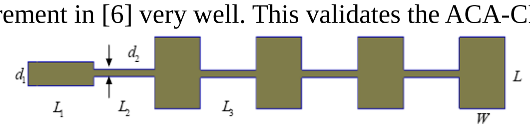

Fig. 1. The geometries of the microstrip.

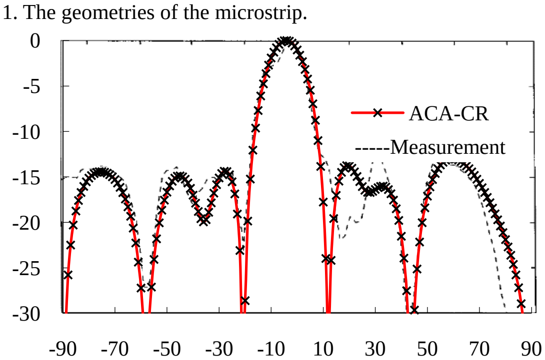

Fig. 2. The radiation pattern of microstrip.

B. The performance of ACA-CR

In order to demonstrate the performance of the ACA-CR, we consider the radiation of a finite array of 32×32 rectangular microstrip patches. The array is on infinite grounded dielectric slab, see Fig. 3. The configuration of the unit of the arrays is depicted in [7]. The number of unknowns in finite array is 33792. The memory requirement and CPU time per MVP for the microstrip are displayed in Table 1. It can be seen that the ACA-CR is superior to original ACA in term of memory requirements and MVP time.

TABLE 1
The comparisons of memory, MVP time and solution time.

Algorithm	Memory(Mb)	MVP time (second)	Solution time (second)
Original ACA	356	0.196	123
ACA-CR	274	0.137	96

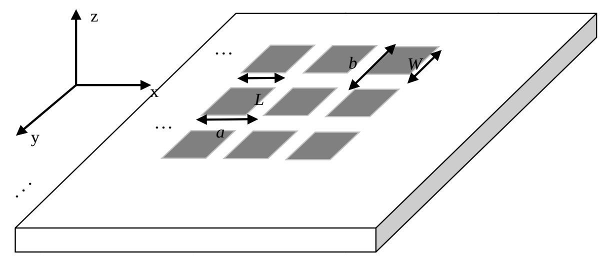

Fig. 3. The geometries of microstrip array. The length and width of the patch is L=3.66cm and W=2.60cm. The gap between the patches is a=b=5.517cm. The dielectric constant and thickness of the substrates is 2.17 and d=0.158cm.

IV. Conclusions

In this paper, an efficient compression representation of ACA algorithm is proposed. Numerical experiments demonstrate that the proposed ACA-CR is superior to original ACA in terms of the MVP time and memory usage.

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